Center of Mass and Collision

Question1

Two blocks of masses in the ratio $m : n$ are connected by a light inextensible string passing over a frictionless fixed pulley. If the system of the blocks is released from rest, then the acceleration of the centre of mass of the system of the blocks is

($g =$ acceleration due to gravity)

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Options:

A.

$\left(\frac{m+n}{m-n}\right)^2 g$

B.

$\left(\frac{m-n}{m+n}\right)^2 g$

C.

$\left(\frac{m+n}{m-n}\right) g$

D.

$\left(\frac{m-n}{m+n}\right) g$

Answer: B

Solution:

For block m (moving down)

$mg - T = ma \qquad \ldots(i)$

For block n (moving up)

$T - ng = na \qquad \ldots (ii)$

From Eqs. (i) and (ii), we get

$$mg - ng = ma + na$$
$$(m - n)g = (m + n)a$$
$$a = \frac{(m - n)}{(m + n)} g$$

$\therefore \quad a_m = +a$ and $a_n = -a$

then, $a_{\text{CM}} = \frac{ma - na}{m + n} = \frac{(m - n)}{(m + n)} \times a$

$a_{\text{CM}} = \left(\frac{m - n}{m + n}\right)^2 g$

Question2

A ball A of mass 1.2 kg moving with a velocity of 8.4 ms^{-1} makes one-dimensional elastic collision with a ball B of mass 3.6 kg at rest. The percentage of kinetic energy transferred by ball A to ball B is

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Options:

A. 25%

B. 50%

C. 75%

D. 60%

Answer: C

Solution:

Given,

Mass of ball, $m_A = 1.2$ kg

Mass of ball, $m_B = 3.6$ kg

Initial velocity of ball, $v_{A_i} = 8.4 \text{ ms}^{-1}$

Initial velocity of ball, $v_{B_i} = 0 \text{ ms}^{-1}$

In a one-dimensional elastic collision, the final velocities (v_{A_f} and v_{B_f}) of the two masses can be calculated using the following formula

$v_{A_f} = \frac{(m_A - m_B) \cdot v_{A_i} + 2 m_B v_{B_i}}{m_A + m_B}$

After putting values, we get

$$v_{A_f} = \frac{(1.2 - 3.6) 8.4}{1.2 + 3.6} = \frac{-2.4 \times 8.4}{4.8}$$

$$v_{A_f} = -4.2 \text{ m/s}$$

Similarly, $v_{B_f} = \dfrac{(m_B - m_A) v_{B_i} + 2 m_A v_{A_i}}{m_A + m_B}$

$$v_{B_f} = \frac{0 + 2 \times 1.2 \times 8.4}{1.2 + 3.6}$$

$$= \frac{2.4 \times 8.4}{4.8}$$

$$V_{B_F} = 4.2 \text{ m/s}$$

Initial kinetic energy of ball A is,

$$\text{KE}_{A_i} = \frac{1}{2} m_A (v_{A_i})^2 = \frac{1}{2} \times 1.2 \times 8.4 \times 8.4$$

$$= 42.33 \text{ J}$$

Final kinetic energy of ball B is,

$$\text{KE}_{B_f} = \frac{1}{2} m_B (v_{B_f})^2$$

$$= \frac{1}{2} \times 3.6 \times 4.2 \times 4.2 = 31.75 \text{ J}$$

The percentage of kinetic energy

transferred from ball A to ball B is,

$$\text{KE}_{\text{transferred}} = \frac{\text{KE}_{A_i}}{\text{KE}'} \times 100\%$$

$$= \frac{31.75}{42.33} \times 100\% = 75\%$$

Question3

A meter scale is balanced on a knife edge at its centre. When two coins, each of mass 9 g are kept one above the other at the 10 cm mark, the scale is found to be balanced at 35 cm . The mass of the meter scale is

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Options:

A. 15 g

B. 30 g

C. 45 g

D. 60 g

Answer: B

Solution:

Let mass of meter scale be m.

Mass of each coin $= 9$ g

Total mass of coin $= 18$ g

Length of meter scale $= 100$ cm

The new balance point is at 35 cm

The torque due to the coins placed at the 10 cm mark

Distance from the new balance point to the coins $= (35 - 10)$cm

$= 25$ cm

Torque due to coins

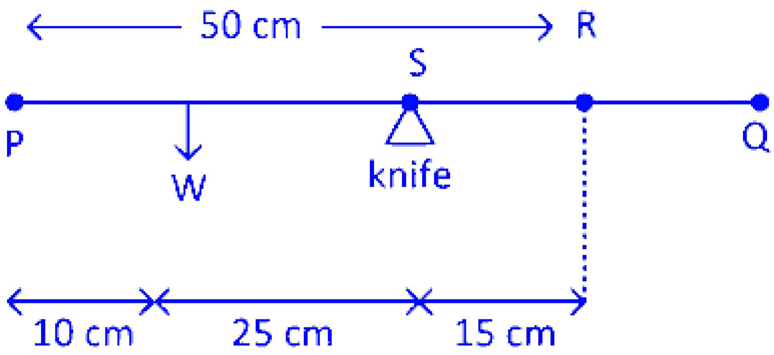
$= F \cdot r = mgr = 18 \times g \times 25$

The centre of mass of the meter scale is at its mid-point (50 cm mark).

Distance from the new balance point to the $= (50 - 35)$cm

Centre of the meter scale $= 15$ cm

Torque due to meter scale $= m \times g \times 15$



For the scale to be balanced, the net torque will conserver. So, from Eq. (i) and Eq. (ii), we get

$18 \times g \times 25 = m \times g \times 15$

$m = \frac{18 \times 25}{15}$

$m = 30 \text{ g}$

Question4

A ball P of mass 0.5 kg moving with a velocity of 10 ms^{-1} collides with another ball Q of mass 1 kg at rest. If the coefficient of restitution is 0.4 , the ratio of the velocities of the balls P and Q after the collision is

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Options:

A. 1 : 7

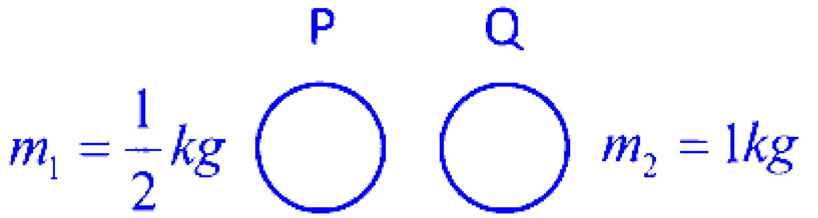
B. 2 : 7

C. 2 : 5

D. 5 : 6

Answer: A

Solution:



$u_1 = 10 \mathrm{~m/s} \quad u_2 = 0 \mathrm{~m/s}$

Coefficient of restitution,

$e = 0.4$

Velocity of m_1 (ball P) after collision is given by

$$v_1 = \left(\frac{m_1 - e m_2}{m_1 + m_2}\right) u_1$$

$$v_1 = \frac{(0.5 - 0.4 \times 1)}{1.5} \times 10$$

$$v_1 = \frac{2}{3} \mathrm{~m/s}$$

Velocity of ball Q (initially rest) after collision

$$v_2 = \left[\frac{m_2 - e m_1}{m_1 + m_2}\right] u_2 + \frac{(1+e) m_1 u_1}{m_1 + m_2}$$

$$v_2 = \frac{(1+e) m_1 u_1}{m_1 + m_2}$$

(as $u_2 = 0$)

$$v_2 = \frac{(1+0.4)(0.5)(10)}{1.5}$$

$$v_2 = \frac{14}{3} \mathrm{~m/s}$$

Ratio of v_1 and v_2

$$\frac{v_1}{v_2} = \frac{2}{3} \times \frac{3}{14} = \frac{1}{7}$$

$v_1 : v_2 = 1 : 7$

Question5

A circular plate of radius r is removed from a uniform circular plate P of radius $4r$ to form a hole. If the distance between the centre of the hole formed and the centre of the plate P is $2r$, then the distance of mass of the remaining portion from the centre of the plate P is

Options:

A. $\frac{r}{3}$

B. $\frac{r}{15}$

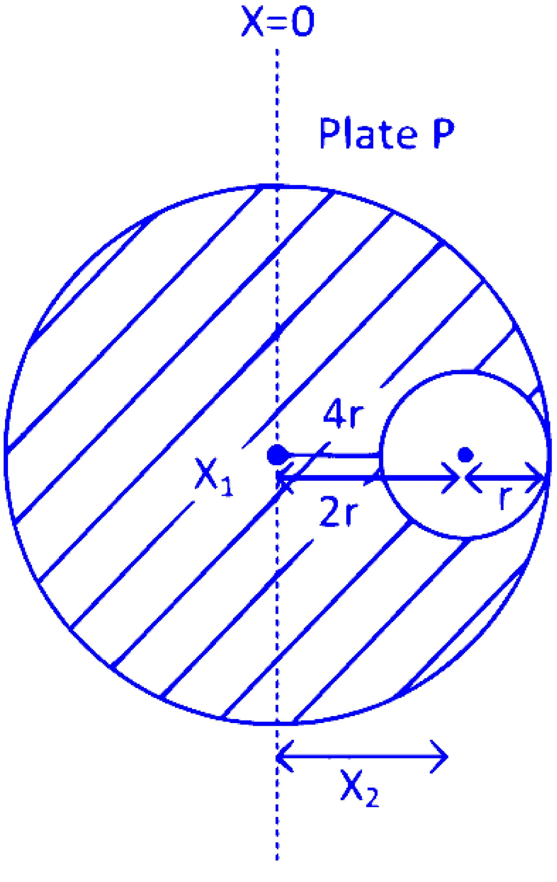
C. $\frac{2r}{15}$

D. $2r$

Answer: C

Solution:

When the section is cut out we use area formula of COM.



$X_{\mathrm{cm}}=\frac{A_1 x_1-A_2 x_2}{A_1-A_2}$

where A_1 area of plate P, A_2 area of cut out portion

$x_1=0, x_2=2r, A_1=\pi(4r)^2, A_2=\pi(r)^2$

$X_{\mathrm{cm}}=\frac{0-\pi r^2 \cdot 2 r}{\pi(4 r)^2-\pi r^2}$

$X_{\mathrm{cm}}=\frac{-\pi r \cdot 2}{\pi 15}$

$X_{\mathrm{cm}}=-\frac{2 r}{15}$

$\text{Distance }=\left|X_{\mathrm{cm}}\right|=\frac{2 r}{15}$

Question6

A ball of mass 1.2 kg moving with a velocity of $12 \mathrm{~ms}^{-1}$ makes one-dimensional collision with anothe stationary ball of mass 1.2 kg . If the coefficient of restitution is $\frac{1}{\sqrt{2}}$, then the ratio of the total kinetic energy of the balls after collision and the initial kinetic energy is

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Options:

A. 3 : 4

B. 1 : 1

C. 2 : 3

D. 3 : $\sqrt{2}$

Answer: A

Solution:

Given,

Mass $m_1=1.2 \mathrm{~kg}, u_1=12 \mathrm{~m} / \mathrm{s} . m_2=1.2 \mathrm{~kg}, u_2=0$

Coefficient of restitution $=1 / \sqrt{2}$

Now apply conservation of linear momentum equation for the collision.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

($v_1, v_2 =$ velocity after collision)

$m[u_1 + u_2] = m[v_1 + v_2]$ [same masses]

as $u_2 = 0, u_1 = 12 \text{ m/s}$

$12 = v_2 + v_1$

From coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow \frac{1}{\sqrt{2}} = \frac{v_2 - v_1}{12}$$

$\Rightarrow 6\sqrt{2} = v_2 - v_1$

Solving Eqs. (i) and (ii)

$v_1 = (6 - 3\sqrt{2}) \text{m/s}$

and

$v_2 = (3\sqrt{2} + 6) \text{m/s}$

Kinetic energy before collision

$= \frac{1}{2}m_1u_1^2 = \frac{1}{2}(m)(144)$

Kinetic energy after collision

$= \frac{1}{2}m\left[v_1^2 + v_2^2\right]$

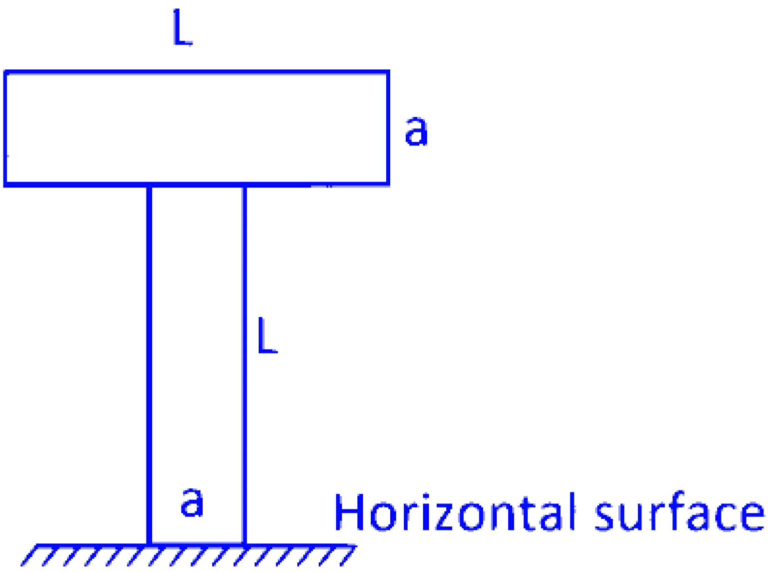
$= \frac{1}{2}m[108]$

Ratio of final (after collision) kinetic energy to initial

$= \frac{\frac{1}{2}m[108]}{\frac{1}{2}m[144]} = \frac{3}{4} = 3 : 4$

Question7

An alphabet T made of two similar thin uniform metal plates of each length L and width a is placed on a horizontal surface as shown in the figure. If the alphabet is vertically inverted, the shift in the position of its centre of mass from the horizontal surface is



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Options:

A. $\frac{L-a}{2}$

B. $\frac{a-L}{2}$

C. $L-\frac{a}{2}$

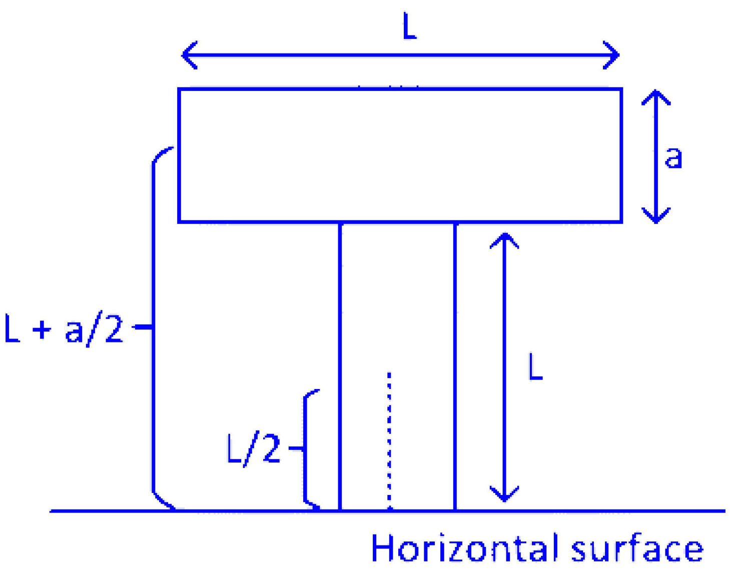
D. $\frac{L}{2}-a$

Answer: A

Solution:

Initial condition,

Centre of mass of two rod, (Y-axis)



$Y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2}$

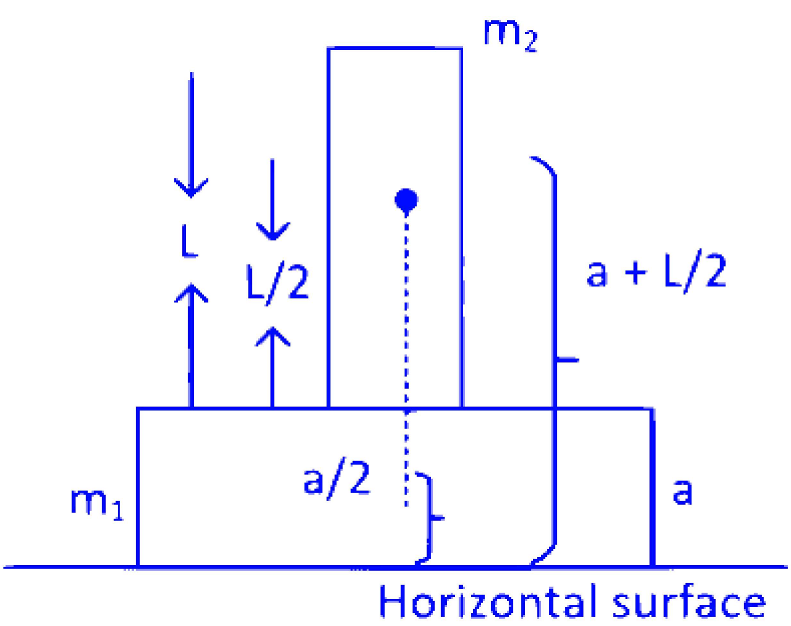
As rod are identical $m_1 = m_2 = m$

$y_1 = \frac{L}{2}, y_2 = L + \frac{a}{2}$

$Y_{cm} = \frac{m\left[\frac{L}{2}+L+\frac{a}{2}\right]}{2m} = \frac{3L+a}{4}$

Final case,

Centre of mass of two rod,



$$Y'_{cm} = \frac{m_1y'_1 + m_2y'_2}{m_1 + m_2}$$

$$y'_1 = \frac{a}{2}, y'_2 = a + \frac{L}{2}$$

$Y'_{cm} = \frac{1}{2}\left[\frac{3a+L}{2}\right] = \frac{3a+L}{4}$

Difference in shift

$Y_{\mathrm{cm}} - Y'_{\mathrm{cm}} = \frac{1}{2}[L - a]$

Question8

Three particles A, B and C of masses m, $2m$ and $3m$ are moving towards north, south and east respectively. If the velocities of the particles A, B and C are $6 \mathrm{~ms}^{-1}$. $12 \mathrm{~ms}^{-1}$ and $8 \mathrm{~ms}^{-1}$ respectively, then the velocity of the centre of mass of the system of particles is

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Options:

A. $7 \mathrm{~ms}^{-1}$

B. $5 \mathrm{~ms}^{-1}$

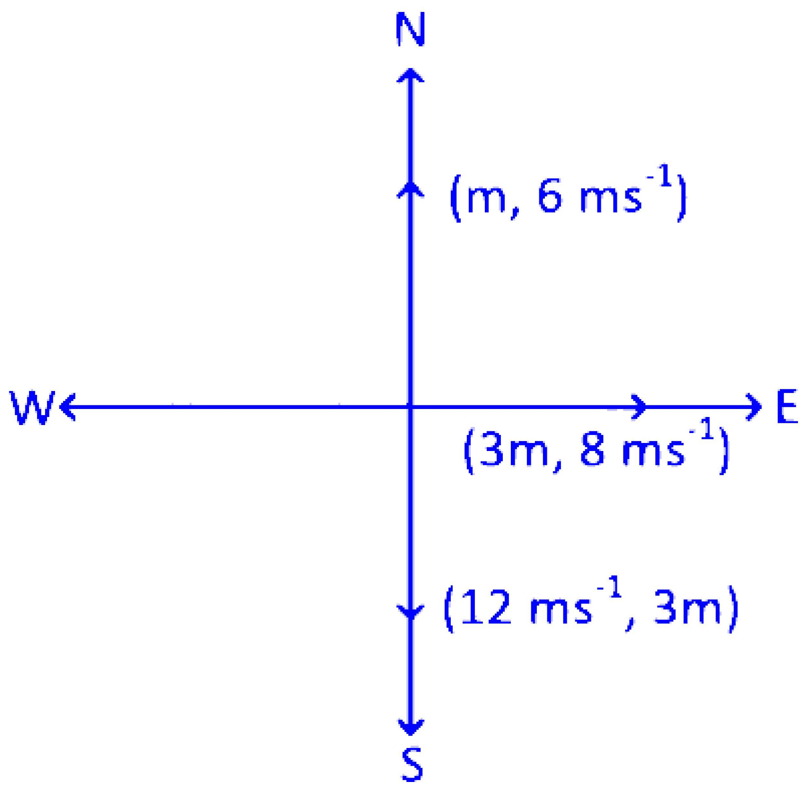
C. $26 \mathrm{~ms}^{-1}$

D. $8 \mathrm{~ms}^{-1}$

Answer: B

Solution:

From the figure, velocity of centre of mass,



$$v_{\text{COM}} = \frac{m \times (6\hat{\mathbf{j}}) + 2m(-12\hat{\mathbf{j}}) + 3m(8\hat{\mathbf{i}})}{m + 2m + 3m}$$

$$= \frac{6m\hat{\mathbf{j}} - 24m\hat{\mathbf{j}} + 24m\hat{\mathbf{i}}}{6m}$$

$$= -3\hat{\mathbf{j}} + 4\hat{\mathbf{i}}$$

Magnitude of velocity of centre of mass

$$= \sqrt{3^2 + 4^2}$$

$$= 5 \text{ m/s}$$

Question9

The sphere A of mass m moving with a constant velocity hits another sphere B of mass $2m$ at rest. If the coefficient of restitution is 0.4 the ratio of the velocities of the spheres A and B after collision is

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Options:

A. 3 : 1

B. 1 : 5

C. 1 : 7

D. 4 : 1

Answer: C

Solution:

Here,

$$m \cdot v_{A_1} = m \cdot v_{B_2} + 2m \cdot v_{B_2}$$

$$v_{B_2} - v_{A_2} = 0.4 \times v_{A_1}$$

$$m \cdot v_{A_1} = m \cdot v_{A_2} + 2m \times (0.4 \times v_{A_1} + v_{A_2})$$

For A

$$v_{A_1} = 3 \cdot v_{A_2} + 0.8 \times v_{A_1}$$

$$v_{A_2} = \frac{0.2}{3} \times v_{A_1}$$

For B

$$v_{B_2} = \frac{0.4 \times 3 + 0.2}{3} \times v_{A_1}$$

$$= \frac{1.4}{3} v_{A_1}$$

The ratio of both masses

$$\frac{v_{A_2}}{v_{B_2}} = \frac{\left(\frac{0.2}{3}\right) \times v_{A_1}}{\left(\frac{1.4}{3}\right) \times v_{A_1}}$$

$$\frac{v_{A_2}}{v_{B_2}} = \frac{1}{7}$$

Question10

Four identical particles each of mass m are kept at the four corners of a square of side a. If one of the particles is removed, the shift in the position of the centre of mass is

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Options:

A. $\sqrt{2a}$

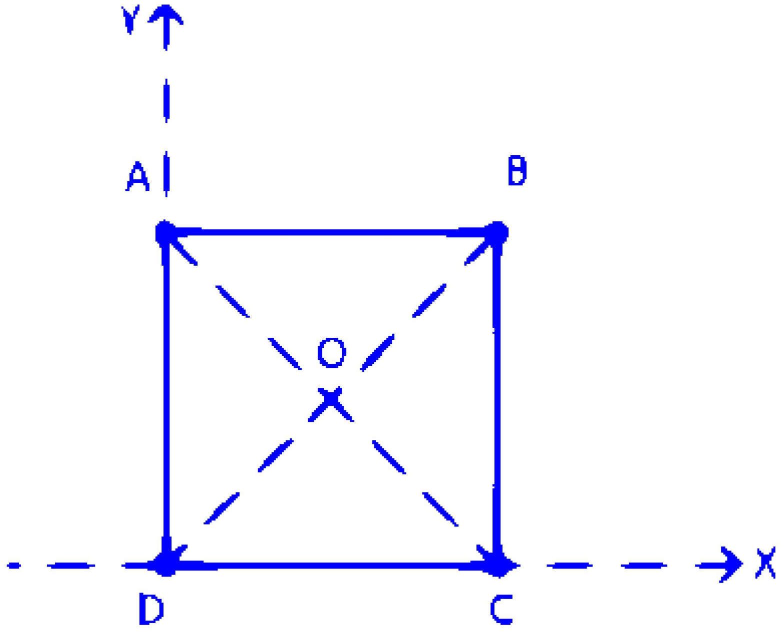
B. $\frac{3a}{\sqrt{2}}$

C. $\frac{a}{\sqrt{2}}$

D. $\frac{a}{3\sqrt{2}}$

Answer: D

Solution:



Original system (with 4 particles)

Let coordinates of the particle as

$A = (0, a)$

$B = (a, a)$

$C = (a, 0)$

$D = (0, 0)$

The centre of mass of system

$$X_{\text{com}} = \frac{mx_A + mx_B + mx_C + mx_D}{4m}$$

$$Y_{\text{com}} = \frac{my_A + my_B + my_C + my_D}{4m}$$

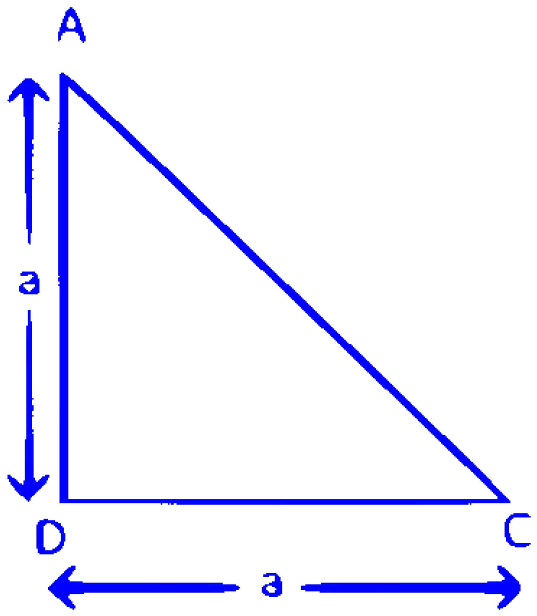
Solving,

$$X_{\text{com}} = \frac{0 + a + a + 0}{4} = \frac{a}{2}$$

$$Y_{\text{com}} = \frac{a + a + 0 + 0}{4} = \frac{a}{2}$$

The centre of mass of original system at $\left(\frac{a}{2}, \frac{a}{2}\right)$

New centre of mass, when one particle (B) is removed,



$$X'_{\text{com}} = \frac{m \cdot x_A + m \cdot x_C + m \cdot x_D}{3m}$$

$$Y'_{\text{com}} = \frac{m \cdot y_A + m \cdot y_C + m \cdot y_D}{3m}$$

$$X'_{\text{con}} = \frac{0 + a + a}{3} = \frac{2a}{3}$$

$$Y'_{\text{com}} = \frac{a + 0 + 0}{3} = \frac{a}{3}$$

Now, shift in position

$$\sqrt{\left(\frac{2a}{3}-\frac{a}{2}\right)^2+\left(\frac{a}{3}-\frac{a}{2}\right)^2}=\frac{a}{3\sqrt{2}}$$

Thus, centre of mass of 3 system shift to $\frac{a}{3\sqrt{2}}$

Question11

A system consists of two particles of masses m_1 and m_2. If the particle of mass m_1 is moved towards the centre of mass through a distance d, then the distance the second particle should be moved, so as to keep the centre of mass at the same position is

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Options:

A. $-\frac{m_2}{m_1}d$

B. $\frac{m_2}{m_1+m_2}d$

C. $-\frac{m_1}{m_2}d$

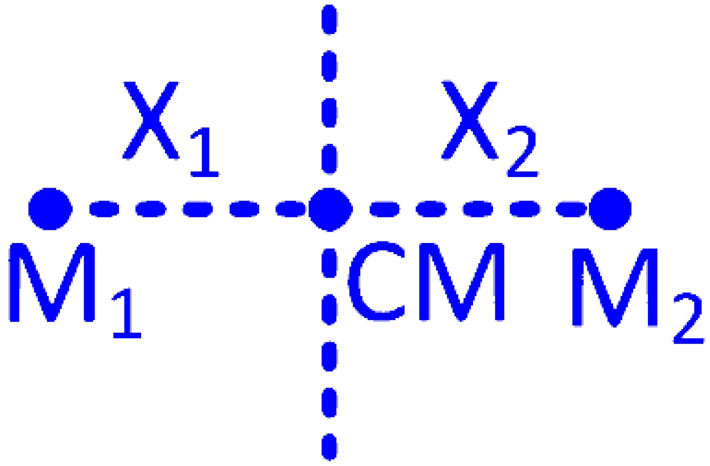
D. $\frac{m_1}{m_2}d$

Answer: C

Solution:

Given,

Two masses m_1 and m_2 moves toward CM by d distance.



We know position of CM

$X_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

Shift in position of centre of mass,

$\Delta x_{\text{CM}} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$

But $\Delta x_{CM} = 0$ (Given)

$$m_1 \Delta x_1 + m_2 \Delta x_2 = 0$$
$$\left(\frac{m_1}{m_2}\right) \Delta x_1 = -\Delta x_2$$
$$\Delta x_2 = -\frac{m_1}{m_2} d$$
